WADHWAN CITY
University (Winter) Examination -2013
Subject Name: -Operational Research
Marks :70
Course Name :M.Sc(Mathematics) Sem-I Duration :- 3:00 Hours

## Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary.
(2) Use of Programmable calculator \& any other electronic instrument is prohibited.
(3) Instructions written on main answer Book are strictly to be obeyed.
(4)Draw neat diagrams \& figures (If necessary) at right places.
(5) Assume suitable \& Perfect data if needed.

## SECTION-I

Q-1 a) Explain general mathematical form of linear programming problem.
b) Define: Artificial variable.
c) Solution by simplex method requires that an LPP should have no negative values in the right hand sides of the constraints. Determine whether statement is true or false.
d) Define: Unbounded solution.
e) For maximization problem, what is coefficient of an artificial variable in the objective function?

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f) Every linear programming problem has a unique solution. Determine whether statement is true or false.
Q-2 a) A company manufactures two type of products $A$ and $B$ and sells them at a profit of Rs-2 on $A$ and Rs-3 on B. Each products is processed on two machines $G$ and $H$. Type $A$ required one minute of processing time on $G$ and two minute of processing time on $H$. Type $B$ required one minute of processing time on $G$ and one minute of processing time on $H$. Then machine $G$ is available for not more than 6 hours 40 minutes, while machine $H$ is available for 10 hours during any working day. Formulate the given problem as linear programming problem.
b) Use the penalty (Big-M) method to solve following LP problem.

$$
\begin{align*}
& \text { Max } z=2 x_{1}+3 x_{2}  \tag{07}\\
& \text { Subject to } \\
& 3 x_{1}+x_{2}=3 \\
& 4 x_{1}+3 x_{2} \geq 6 \\
& x_{1}+2 x_{2} \leq 4 \\
& \text { and } x_{1}, x_{2} \geq 0 .
\end{align*}
$$

## OR

Q-2 a) A city hospital has the following minimum daily requirement for nurses.

| Period | Clock time | Minimum no of nurses required |
| :---: | :---: | :---: |
| 1 | 06 a.m. to 10 a.m. | 02 |
| 2 | 10 a.m. to 02 p.m. | 07 |
| 3 | 02 p.m. to 06 p.m. | 15 |
| 4 | 06 p.m. to 10 p.m. | 18 |
| 5 | 10 p.m. to 02 a.m. | 20 |
| 6 | 02 a.m. to 06 a.m. | 06 |

Nurses report to the hospital at the beginning of each period and work for 8 successive hour. The hospital wants to determine the minimum number of nurses to be employed. So that there will be sufficient no. of nurses available for each period. Formulate this as linear programming problem by setting appropriate constraints and objective function.
b) Use the graphical method to solve the following LP problem.
$\operatorname{Max} z=10 x_{1}+15 x_{2}$
Subject to

$$
\begin{gathered}
2 x_{1}+x_{2} \leq 26 \\
2 x_{1}+4 x_{2} \leq 56 \\
-x_{1}+x_{2} \leq 5 \\
\text { and } x_{1}, x_{2} \geq 0 .
\end{gathered}
$$

Q-3 a) Write dual to the following LP problem
i) $\operatorname{Max} z=x_{1}-x_{2}+3 x_{3}$
ii) $\operatorname{Min} z=x_{1}+2 x_{2}$

$$
\begin{align*}
& \text { Subject to } \\
& \qquad \begin{array}{l}
x_{1}+x_{2}+x_{3} \leq 10 \\
2 x_{1}-x_{3} \leq 2 \\
2 x_{1}-2 x_{2}-3 x_{3} \leq 6 \\
\text { and } x_{1}, x_{2}, x_{3} \geq 0
\end{array}
\end{align*}
$$

Subject to

$$
\begin{aligned}
2 x_{1}+4 x_{2} & \leq 160 \\
x_{1}-x_{2} & =30 \\
x_{1} \quad & \geq 10 \\
\text { and } x_{1}, x_{2} & \geq 0
\end{aligned}
$$

b) Discuss simplex method. If possible solve the following LP problem by using simplex method.
$\operatorname{Max} z=3 x_{1}+2 x_{2}$
Subject to

$$
\begin{aligned}
-2 x_{1}+3 x_{2} & \leq 9 \\
x_{1}-5 x_{2} & \geq-20
\end{aligned}
$$

and $x_{1}, x_{2} \geq 0$.
OR


Q-3 a) Use duality problem, solve following LP problem.

$$
\begin{equation*}
\operatorname{Min} z=4 x_{1}+2 x_{2} \tag{07}
\end{equation*}
$$

Subject to

$$
\begin{aligned}
& 3 x_{1}+x_{2} \geq 27 \\
& x_{1}+x_{2} \geq 21 \\
& \text { and } \quad x_{1}, x_{2} \geq 0
\end{aligned}
$$

b) Use the simplex method to solve following LP problem.

$$
\begin{equation*}
\operatorname{Max} z=3 x_{1}+5 x_{2}+4 x_{3} \tag{07}
\end{equation*}
$$

Subject to

$$
2 x_{1}+3 x_{2} \leq 8
$$

$$
2 x_{2}+5 x_{3} \leq 10
$$

$$
3 x_{1}+2 x_{2}+4 x_{3} \leq 15
$$

and $x_{1}, x_{2}, x_{3} \geq 0$.

## SECTION-II

Q-4 a) Write general non-linear programming problem.
b) Define: Feasible solution of a transportation problem.
c) The occurrence of degeneracy while solving actransportation problem means that the solution so obtained is not feasible Determine whether statement is true or false.
d) Maximization assignment problem is transformed in to a minimization
problem by subtracting each entry in a column from the maximum value in that column. Determine whether statement is true or false.
e) An assignment problem can be solved by simplex and transportation problem. Determine whether statement is true or false.
f) The solution to a transportation problem with $m$-rows and $n$-columns is feasible if number of positive allocations are $m+n+1$. Determine whether statements is true or false.

Q-5 a) Write the general mathematical model of a Assignment Problem. Solve the minimal assignment problem whose effectiveness matrix is given below.

|  | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| I | 2 | 3 | 4 | 5 |
| II | 4 | 5 | 6 | 7 |
| III | 7 | 8 | 9 | 8 |
| IV | 3 | 5 | 8 | 4 |

b) A steel company has three furnaces and five rolling mills, transportation cost (rupees per quintal) for shipping steel form furnaces to rolling mills are as show in the following table. Determine the initial basic feasible solution by north-west corner method.

| Furnaces | Mills |  |  |  |  | Capacity |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | M1 | M2 | M3 | M4 | M5 |  |
| F1 | 4 | 2 | 3 | 2 | 6 | 8 |
| F2 | 5 | 4 | 5 | 2 | 1 | 12 |
| F3 | 6 | 5 | 4 | 7 | 3 | 14 |
| Requirement | 4 | 4 | 6 | 8 | 8 |  |

OR
Q-5 a) Five operator $A, B, C, D, E$ have to be assign to five machine $1,2,3,4,5$ the assignment cost are given in the following table operator $A$ can not operator machine $3, C$ can not operator machine 4 . Find the optimal assignment.

|  | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 5 | 5 | $\infty$ | 2 | 6 |
| B | 7 | 4 | $H 2 N /$ | 3 | 4 |
| C | 9 | 3 | 5 | 0 | 3 |
| D | 7 | 2 | 6 | 7 | 2 |
| E | 6 | 5 |  | 9 | 1 |

b) Write the general mathematical model of a Transportation Problem. Determine the initial basic feasible solution to the following transportation problem using Vogel's Approximation Method.

|  | A | B | C | D | Available |
| :---: | :---: | :---: | :---: | :---: | :---: |
| P | 10 | 7 | 3 | 6 | 3 |
| Q | 1 | 6 | 7 | 3 | 5 |
| R | 7 | 4 | 5 | 3 | 7 |
| Requirement | 3 | 2 | 6 | 4 | 15 |

Q-6 a) Use dual simplex method to solve the LP problem
$\operatorname{Min} z=3 x_{1}+x_{2}$
Subject to

$$
\begin{gathered}
x_{1}+x_{2} \geq 1 \\
2 x_{1}+3 x_{2} \geq 2
\end{gathered}
$$

and $x_{1}, x_{2} \geq 0$
b) Solve the following transportation problem using UV Method.

| Origins | Destinations |  |  | Availability |
| :---: | :---: | :---: | :---: | :---: |
|  | $D_{1}$ | $D_{2}$ | $D_{3}$ |  |
| $O_{1}$ | 2 | 7 | 4 | 5 |
| $O_{2}$ | 3 | 3 | 7 | 8 |
| $O_{3}$ | 5 | 4 | 1 | 7 |
| $O_{4}$ | 1 | 6 | 2 | 14 |
| Requirement | 7 | 9 | 18 | 34 |

Q-6 a) Use Wolfe's method to solve the quadratic programming problem
$\operatorname{Max} z=4 x_{1}+6 x_{2}-2 x_{1}^{2}-2 x_{1} x_{2}-2 x_{2}^{2}$
Subject to

$$
x_{1}+2 x_{2} \leq 2
$$

and

$$
\begin{equation*}
x_{1}, x_{2} \geq 0 \tag{07}
\end{equation*}
$$

b) Determine the initial basic feasible solution to the following transportation problem using (i) Column-minima method/; (ii) Least cost method

|  | A | B | C | D | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| P | 5 | 3 | 6 | 4 | 30 |
| Q | 3 | 4 | 7 | 8 | 15 |
| R | 9 | 6 | 5 | 8 | 15 |
| Demand | 10 | 25 | 18 | 7 | 60 |

